Czech Summer School on Stochastic Geometry

Institute of Mathematics and Biomathematics, Faculty of Science,
University of South Bohemia, České Budějovice, Czech Republic

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21. - 24. 6. 2010

The book of Abstracts

Random marked sets

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Abstract: Random marked sets were introduced by Ballani et al.(2009) as a general model in stochastic geometry. Their basic special cases are marked point processes or level sets of random fields. We will investigate also other types of random marked sets, their second order characteristics and the hypothesis of a random-field model.

References

F. Ballani, Z. Kabluchko, M. Schlather, Random marked sets. (2009) arXiv:0903.2388v1 [math.PR]

Applications of the random field theory in medical imaging

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Abstract: Excursion set of a smooth random field is the set of points where the field exceeds a given threshold. Euler characteristic counts the number of connected components in such a set minus the number of "holes". One of the interesting results of the random field theory is that (under some assumptions) the expected value of the Euler characteristic of the excursion set is given by an explicit formula.
This formula can be used to determine the areas with significant "activation" in some types of medical images. In the talk the properties of real as well as simulated PET (Positron Emission Tomography) images will be discussed and the applications of the random field theory in detection of liver metastases and optimization of the injection dose will be presented.

References

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Worsley, K.J. (1997). The geometry of random images. *Chance*, **9**, 27-40.

On an evaluation of neurophysiological data

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Abstract: We consider experimental data where a rat is searching for food in an arena and the activity of its neuron is recorded. In comparison with Frcalová et al. (2010) further statistical methods are used. We develop a test of a random-field model (Ballani at al., 2009) in the situation when the random set is the track of the rat and the random field is the driving intensity function of spikes in the Cox point process model.

References

F. Ballani, Z. Kabluchko, M. Schlather, Random marked sets. (2009) arXiv:0903.2388v1 [math.PR]

B. Frcalová, V. Beneš, D. Klement, Spatio-temporal point process filtering methods with an application. Environmetrics 21 (2010), 240-252.

On covariance matrices associated with Poisson hyperplane processes and some geometric inequalities

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Abstract: Among others, it has been proved in Heinrich(2009a) that the total number of vertices $\Psi_V(\varrho\, K)$ of convex polytopes generated by a motion-invariant Poisson hyperplane process in an expanding region $\varrho\, K$, where $K \subset \mathbb{R}^d$ denotes a fixed convex body with positive d-volume $\nu_d(K)$, is asymptotically normally distributed with mean $(\lambda\, \varrho\, \kappa_{d-1}/d\, \kappa_d)^d\, \nu_d(K)$ and a variance being asymptotically equal to $(d-1)\, \kappa_{d-1}^2\, (\lambda\, \varrho\, \kappa_{d-1}/d\, \kappa_d)^{2d-1}\, J_d(K)$ as $\varrho \to \infty$. Here, λ gives the mean total $(d-1)$-volume of the hyperplanes in the unit cube and κ_d stands for the d-volume of the unit ball, whereas $J_d(K)$ denotes the dth-order chord power integral of K which reflects the influence of the shape of K on the variance of $\Psi_V(\varrho\, K)$. In the talk we also study the asymptotic behaviour of the number of $(d-k)$-intersection flats hitting $\varrho\, K$ as well as their total $(d-k)$-volume within $\varrho\, K$ for $k = 1, ..., d$. Furthermore, we derive tight lower and upper bounds of $J_d(K)$ for full-ellipsoids in $\mathbb{R}^d$ and discuss the validity of these inequalities for any convex full-body K. In the planar case we will consider some special convex discs supporting the conjecture, see Heinrich (2009b), that

$$\frac{32}{3}\, \frac{(\,\nu_2(K)\,)^2}{\nu_1(\partial\, K)} \le J_2(K) = \int_K \int_K \frac{\mathrm{d}x\, \mathrm{d}y}{\|\, x - y\,\|} \qquad \text{for any convex discs } K \text{ in } \mathbb{R}^2\,.$$

References

Heinrich, L. (2009a). CLTs for motion-invariant hyperplanes in expanding convex bodies. *Rendiconti del Circolo Matematico di Palermo*, Ser. II, Suppl. **81**, 187-212.

Heinrich, L. (2009b). On lower bounds of second-order chord power integrals of convex discs. *Rendiconti del Circolo Matematico di Palermo*, Ser. II, Suppl. **81**, 213-222.

Estimating parameters in Quermass-interaction process

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Abstract: Consider a random set observed in a window $W \subset \mathbf{R}^2$. The set is given by a union of interacting discs with randomly scattered centers and with arbitrary (random or deterministic) radii. Assume that its probability measure is given by a density with respect to the probability measure of a stationary Boolean model, i.e. with respect to a process of discs without any interactions, whose centers form a stationary Poisson point process with an intensity ρ. Next, assume that the density is of the form

$$f_\theta(\mathbf{x}) = \frac{\exp\{\theta_1 A(U_\mathbf{x}) + \theta_2 L(U_\mathbf{x}) + \theta_3 \chi(U_\mathbf{x})\}}{c_\theta}$$

for any finite configuration of discs $\mathbf{x}$, where $A(U_\mathbf{x})$ is the area, $L(U_\mathbf{x})$ the perimeter and $\chi(U_\mathbf{x})$ the Euler-Poincare characteristic of the union $U_\mathbf{x}$ composed of the discs from the configuration $\mathbf{x}$. Further, $\theta = (\theta_1, \theta_2, \theta_3)$ is a vector of parameters and c_θ denotes a normalizing constant. Such a model is called Quermass-interaction process.

In this contribution, a method for estimating the parameters $\theta_1, \theta_2, \theta_3$ and ρ based on Takacs-Fiksel procedure will be described and briefly compared with MCMC maximum likelihood method.

Almost sure convergence of the surface area of Wiener sausage

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Abstract: We study limit behaviour of surface area of r-neightbourhood of bounded set in $\mathbb{R}^d$. For known limit behaviour of its volume we postulate limit behaviour of its surface area. Our results in dimension $d \geq 3$ are applied for almost surely asymptotics of surface area of Wiener sausage. These results correspond with weaker convergence of mean volume of Wiener sausage. For $d = 2$ it is not possible to use the proposed technique, we can demonstrate it by a simple counterexample.

References

Rataj, J., Schmidt, J.V., Spodarev, E. (2009). On the expected surface area of the Wiener sausage. *Math. Nachr.* **282** (2009), 591–603.

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Kernel estimation of mean densities for inhomogeneous random closed sets

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Abstract: Random closed sets of different Hausdorff dimensions can be a model for many real phenomena. The problem of estimation of mean length (surface or more generally $\mathcal{H}^k$) intensity is widely studied for homogeneous case. For inhomogeneous processes one wants to estimate the mean length ($\mathcal{H}^k$) density $\lambda(x)$. A non-parametric estimation method is presented here, the kernel method, which is usually used for smoothening density estimations for random variables and mean intensity estimations of an inhomogeneous point processes. This approach is then generalized to random $\mathcal{H}^k$-rectifiable sets. At first the general case is discussed. Then a concrete formula for numerical estimation of $\lambda(x)$ is derived for fiber process in $\mathbb{R}^2$. A few examples of estimation for simulated data are also included.

References

Beneš, V. and Rataj, J. (2004). *Stochastic Geometry: Selected topics.* Kluwer academic publishers.

Ambrosio, A., Capasso, V. and Villa, E. (2009). On aproximation of mean densities of random closed sets. *Bernoulli* , **Volume 15, Number 4 (2009)**, 1222-1242.

Discrete analogues of stable distributions and their properties

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Abstract: The main difficulty for a definition of stability property in the class of integer-valued random variables is that it is impossible to make ordinary normalization among this class. Some variants of random normalization will be considered in details. The class of all possible normalization procedures can be described in algebraic terms. Different normalization procedures lead to different notions of stability for integer-valued random variables. Classical stable distributions can be obtained from their discrete analogues by passing to a limit. It appears that the discrete normal distribution posses many properties of the classical one, including well-known H. Cramér Theorem.

Testing mammary tissues on the Boolean model assumption

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Abstract: The methods for testing the Boolean model assumption from binary images are summarized. The Monte Carlo method based on the approximation of the envelops by multi-normal distribution with the normalized intrinsic volumes densities of parallel sets as a summary statistics is chosen with respect to its biggest power. The binary images of mammary cancer tissues and mastopathic tissues are tested on the Boolean model assumption. It is proven that the mastopathic tissues deviates from the Boolean model significantly more than mammary cancer tissues.

Local stereology and extremes

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Abstract: Stereology is concerned with drawing inference about particle population from lower dimensional sections of the particles. Local stereological methods form quite modern branch of stereology, see (Vedel Jensen, 1998) for a comprehensive exposition. They require that we can associate a reference point to each particle and accomplish sectioning through this reference point (local probes). One of the aims in stereology is to estimate extremes of particle parameters from the observation of test probes of lower dimension. This field is referred to as stereology of extremes, see (Beneš and Rataj, 2004). So far in the literature, only isotropic uniform random probes were considered. We review the existing results and discuss the possibility to use local probes.

References

Beneš, V. and Rataj, J. (2004). *Stochastic Geometry: Selected Topics.* Boston: Kluwer Academic Publishers.

Vedel Jensen, E. B. (1998). *Local Stereology.* Singapore: World Scientific.

Moment estimation for inhomogeneous spatial Cox processes

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Abstract: In the paper we give an overview of the available moment estimation methods for parametric inhomogeneous spatial Cox process models. These became very popular recently as alternatives to the maximum likelihood estimation. The reason is that ML estimation for Cox processes is computationaly very demanding since it needs an enormous amount of MCMC simulations. We first give an overview of the moment estimation methods for the homogeneous case and then review the recent developments for the inhomogeneous case. The reason is that all the methods for the inhomogeneous case are based on a 2-step approach. They combine the Poisson likelihood for estimation of inhomogeneity with generalizations of the homogeneous case methods for estimation of interaction. The second step is made conditionally on the already obtained estimates of the inhomogeneity parameters.

Curvatures and integral geometry

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Abstract: We consider the following type of valuations on the set of convex bodies in $\mathbb{R}^d$ (or, more generally, on the space of sets with positive reach, or on even larger families of "bodies"). We intersect a body X with a j-flat, take the total kth curvature measure (kth intrinsic volume for convex bodies) of the intersection, and integrate w.r.t. all translations/rotations of the sectioning flat. There are explicit integral formulae for such valuations. We show that they can be expressed as integrals of certain functions (independent of the body X) with respect to s.c. extended curvature measures, which are measures defined by integrating of generalized principal curvatures and supported by the triples (x, n, V), x being a boundary point of x, n a unit outer normal vector at x, and V a linear subspace of given dimension perpendicular to n. We remark that the integrated function does depend on the whole triple, i.e., a reduction to classical curvature or support measures is not possible. We shall discuss some applications and mention certain relations to other problems.

Perfect simulation in stochastic geometry

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Abstract: Perfect simulation converts MCMC algorithms into algorithms which return exact draws from the target distribution, instead of approximations based on long-time convergence to equilibrium. In recent years a lot of various perfect simulation algorithms were developed. Some of these methods are discussed and applied to Strauss, Area-interaction and certain cluster point processes. Described algorithms and their properties are compared theoretically and also by a simulation study.

References

Brix, A., and Kendall, W. (2002). *Simulation of cluster point processes without edge effects. Advances in Applied Probability*, **34**, 267-280.

Hggstrm, O., van Lieshout, M.N.M., and Møller, J. (1998). *Characterization results and Markov chain Monte Carlo algorithms including exact simulation for some spatial point processes. Bernoulli*, **5**, 641-659.

Kendall, W.S., and Møller, J. (2000). *Perfect simulation using dominating processes on ordered state spaces, with application to locally stable point processes. Advances in Applied Probability*, **32**, 844 - 865.

An Application of Particle Filter to a Point Process of Discs

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Abstract: Particle filter is a sequential algorithm based on Monte-Carlo methods, see Doucet et al (2001), which can be used to evaluate the posterior density of the state parameter θ of a Markov Chain.

Consider a point process of random discs in a bounded subset of R^2 with a density p w.r.t. a reference Poisson point process of discs, see Møller, Helisová (2008). Let p be from the exponential family and $\theta \in R^4$ be a parameter of this density. Assume that θ is developed in time as a Markov Chain given by the following state equation

$$\theta_t = \theta_{t-1} + \eta_t, \qquad t = 1, \ldots, n.$$

Here θ_0 is fixed and $\eta_t \sim \mathcal{N}(a, \sigma^2)$, $a \in R^4$ and $\sigma^2 > 0$. The aim is to estimate the parameter θ from simulated data using the particle filter.

References

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